

Queen Mary, University of London
MAS317 Linear Algebra II, 2007
Coursework 2

This coursework should be handed in by 1.30 pm on MONDAY October 8th in the orange box on the second floor in the mathematics building.

Late coursework will not be accepted. Write your name and student number at the top and clip all the sheets together.

$$\text{Let } A = \begin{pmatrix} 1 & 3 & -2 \\ 3 & 0 & 3 \\ -2 & 3 & 7 \end{pmatrix} \text{ and } B = \begin{pmatrix} 0 & 2 & 2 \\ 2 & 0 & -1 \\ 2 & -1 & 0 \end{pmatrix}.$$

1. **[20 marks]** Use Method B, to find a diagonal matrix D over $\mathbb{Q}$ and an invertible matrix P over $\mathbb{Q}$ such that $D = P^T A P$ by applying the method of completing the square to the quadratic form:

$$q(x_1, x_2, x_3) = x_1^2 + 6x_1x_2 - 4x_1x_3 + 6x_2x_3 + 7x_3^2.$$

(Note that this quadratic form has matrix equal to A .)

2. **[30 marks]** Work out how to repeat question 1 with the matrix B instead of A throughout.
3. **[25 marks]** Use Method C to find a diagonal matrix D over $\mathbb{Q}$ and an invertible matrix P over $\mathbb{Q}$ such that $D = P^T A P$ by finding an orthogonal basis for $\mathbb{Q}^3$ w.r.t. the bilinear form f given by

$$f((x_1, x_2, x_3), (y_1, y_2, y_3)) = x_1y_1 + 3x_1y_2 - 2x_1y_3 + 3x_2y_1 + 3x_2y_3 - 2x_3y_1 + 3x_3y_2 + 7x_3y_3.$$

(Note that this bilinear has matrix equal to A with respect to the standard basis for $\mathbb{Q}^3$.)

Note: you should just follow the instructions in the lectures for solving Problem C. You need not prove that this gives a basis -later we will prove that the orthogonal vectors constructed in this way do form a basis for $\mathbb{Q}^3$.

4. **[25 marks]** Work out how to repeat question 3 with the matrix B instead of A throughout.